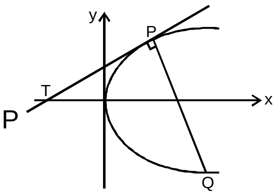
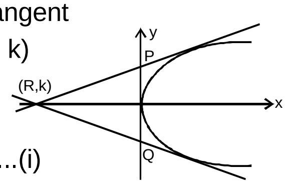
## EXERCISE – IV HINTS & SOLUTIONS

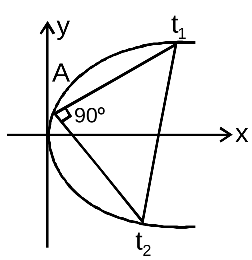
**Sol.1** Point P is (4a, 4a)
equation of tangent
at P :
$x - 2y + 40 = 0$ .....(i)
& equation of normal at P
$2x + y = 12a$ .....(ii)
point T = (–4a, 0)
Q = (9a, –6a)
PT : PQ = 4 : 5



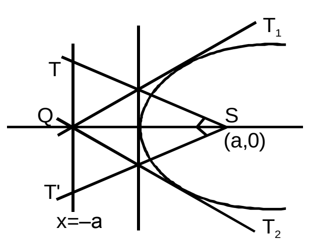

**Sol.2** Equation of pair of tangent
through the point (h, k)
$SS_1 = T$
$\Rightarrow (y^2 - 8x)(k^2 - 8h)$
$= (ky - 4(x + h))^2$ ......(i)
solving with y-axis i.e. x = 0
$-8hy^2 + 8khy - 16h^2 = 0$ d
Now $|y_1 - y_2| = 4 = d$
locus $\Rightarrow$ $y^2 = 8(x + 2)$




**Sol.3**




**Sol.4**




**Sol.5** Let the equation of tangent to $y^2 = 4ax$ is
$y = mx + a/m$ .......(i)
then let the equation that is tangent to y
$x^2 = 4by$ has slope $-1/m$
so equation of tangent to $x^2 = 4by$

$$y = -\frac{1}{m}x - \frac{1}{m^2}b \quad .......(ii)$$

let the point of intersection of (i) & (ii) is (h, k)
so locus ; $(ax + by)(x^2 + y^2) + (bx - ay)^2 = 0$.

**Sol.6** equation of chord through $t_1$ & $t_2$
$2x - (t_1 + t_2($ t $2at_1t_2 = 0$
Now slope = 1

$$\Rightarrow \frac{2}{t_1 + t_2} = 1$$

$\Rightarrow$ $t_1 + t_2 = 2$ (H.P.) ...(i)
point of intersection of normals at $t_1$ & $t_2$
$(h, k) = (a(t_1^2 + t_2^2 + t_1t_2 + 2), -at_1t_2(t_1 + t_2))$
so locus is $2x - y = 12$

**Sol.7** Let the sides of $\Delta$ touches $y^2 = 4ax$, so their equation are $y = m_1x + \frac{a}{m_1}$, $y = m_2x + \frac{a}{m_2}$ &

$$y = m_3x + \frac{a}{m_3} \text{ vertices of } \Delta = \left(\frac{a}{m_1m_2}, a\left(\frac{1}{m_1} + \frac{1}{m_2}\right)\right)$$

$$\left(\frac{a}{m_2m_3}, a\left(\frac{1}{m_2} + \frac{1}{m_3}\right)\right) \;\&\; \left(\frac{a}{m_3m_1}, a\left(\frac{1}{m_1} + \frac{1}{m_3}\right)\right)$$

Let two of these lie one $x^2 = 4by$, then we have to prove that remaining lies on the curve.

$$\text{for two vertices ; } \frac{a^2}{(m_1m_2)^2} = 4ab\left(\frac{1}{m_1} + \frac{1}{m_3}\right) \text{...(ii)}$$

$$\& \quad \frac{a^2}{(m_2m_3)^2} = 4ab\left(\frac{1}{m_2} + \frac{1}{m_3}\right) \text{ .....(ii)}$$

subtracting (i) & (ii) & using (i)

$$\frac{a^2}{(m_3m_1)^2} = 4ab\left(\frac{1}{m_3} + \frac{1}{m_1}\right)$$

(similar to (i) & (ii)
so 3rd vertex also lie on the $x^2 = 4by$

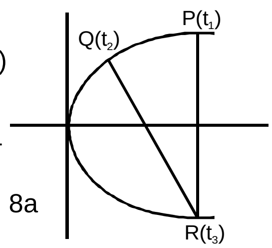
**Sol.8** $2ay + y_1x = 2ay_1 + x_1y_1$ .......(i)
at $(x_2, y_2)$ : $2ay + y_2x = 2ay_2 + x_2y_2$ .......(ii)
at $(x_3, y_3)$ : $2ay + y_3x = 2ay_3 + x_3y_3$ .......(iii)
(i), (ii) & (iii) one concurrent so

$$\begin{vmatrix} 2a & y_1 & 2ay_1 + x_1y_1 \\ 2a & y_2 & 2ay_2 + x_2y_2 \\ 2a & y_3 & 2ay_3 + x_3y_3 \end{vmatrix} = 0$$

**Sol.9** Let e know that

$t_1 + t_2 + t_3 = 0$ ....(i)

& $t_1t_2 = 2$ ....(ii)




$\Rightarrow t_3^2 = \dfrac{4}{t_1^2} + t_1^2 + 4$

$\Rightarrow t_3^2 > 8 \Rightarrow at_3^2 > 8a$

$\Rightarrow$ abscissa > 8a

**Sol.10** Let P is $(at^2, 2at)$

equation of nromal

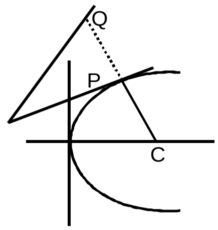
$y + tx = 2at + at^3$

so C = $(2a + at^2, 0)$

Let Q is (h, k).

using mid point formula

so Q = $(at^2 – 2a, 4at)$




so Locus of (h, k) $\Rightarrow$ $16ah = k^2 – 3za^2$

$\Rightarrow y^2 = 16a (x + 2a)$

Now

equation of tangent at $(at^2, 2at)$ to $y^2 = 4ax$

$ty = x + at^2$ ........(i)

equation of tangent at

$(at^2 – 2a, 4at)$ to $y^2 = 16a(x + 2a)$

$ty = 2x + 2at^2 + 4a$ ........(ii)

point of intersection of (i) & (ii) = $\left(–at^2 – 4a, –\dfrac{4a}{t}\right)$

Locus of point of intersection = (h, k) let

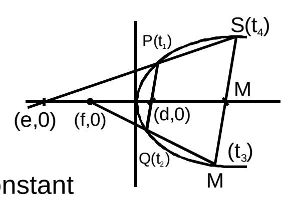
$\Rightarrow y^2(x + 2a) + 16a^3 = 0.$

**Sol.11** Given

$t_1t_2 = – d/a$ ....(i)

$t_2t_3 = – f/a$ ....(ii)

$t_1t_4 = – e/a$ ....(iii)




Now $t_3t_4 = –\dfrac{fe}{c/a}$ = constant

so fourth side also passes through a fixed point.

**Sol.12** Center of circle = (20, 8)

which is outside the parabola

Point Q is (4, –8) of diameter

(If (36, 24) is one point & (20, 8) is center then using it other point of diameter is (4, –8) & so on)

**Sol.13** Equation of tangent to parabola $y^2 = 4ax$

$y = mx + a/m$ ......(i)

solving with circle $x^2 + y^2 = r^2$

$x^2 (1 + m^2) + 2ax + a^2/m^2 – r^2$ ......(ii)

Let the mid point of P & Q is (h, k)

$h = \dfrac{(x_1 + x_2)}{2} = \dfrac{–a}{1+m^2}$ ......(iii)

$\{\because x_1$ & $x_2$ are roots of (ii)$\}$

Similarly $k = \dfrac{m(m^2+1)}{4a}$ ......(vi)

using (iii) & (iv) locus of (h, k) is $x(x^2 + y^2) + ay^2 = 0$

**Sol.14** Let vertices of the triangle be $t_1$, $t_2$ & $t_3$

so & controid is (h, k)

Now $h = \dfrac{a(t_1^2 + t_2^2 + t_3^2)}{3}$ .....(i)

& $k = \dfrac{2a(t_1 + t_2 + t_3)}{3}$ .....(ii)

also if slope of one side is $\dfrac{2}{t_1 + t_2}$ = $m_2$ (let) &

$\dfrac{2}{t_2 + t_3}$ = $m_2$ (let)

then $\tan 60º = \dfrac{m_2 - m_1}{1+m_1m_2}$ .....(iii)

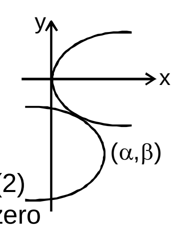
using (i), (ii) & (iii), locus of controid is

$9y^2 – 4ax + 32a^2$

**Sol.15** If the vertex of the moving parabola is $(\alpha, \beta)$ then its equation is

$(y – \beta)^2 = – 4a (x – \alpha)$ ....(1)

solving with $y^2 = 4ax$




$\Rightarrow y^2 – 2\beta y + \beta^2 = – y^2 + 4a\alpha$

$\Rightarrow 2y^2 – 2\beta y + \beta^2 – 4a\alpha = 0$ ....(2)

Discriminant of equation (2) is zero as they touch each other

so $(–2\beta)^2 – 4.2 (\beta^2 – 4a\alpha) = 0 \Rightarrow \beta^2 = 8a\alpha$

locus of $(\alpha, \beta)$ is $y^2 = 8ax$

**Sol.16** Equation of normal at

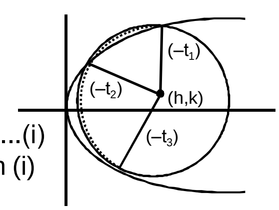
$y + tx = 2at + at^3$

(h, k)

$at^3 + (2a – h)t – k = 0$ ....(i)

If $t_1,t_2,t_3$ one roots of equation (i)

then $\Sigma t_1 = 0$ .....(ii)




$\Sigma t_1t_2 = \dfrac{2a - h}{a}$ .....(iii)

$t_1t_2t_3 = k/a$ .....(iv)

**Sol.17** Slope of AB = – t